"Maxwell's Method of Physical Analogy and the Unreasonable Effectiveness of Mathematics"

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Eugene Wigner, in his classic paper "The Unreasonable Effectiveness of Mathematics in the Natural Sciences," poses two challenges: The first concerns the subject of his title most directly, namely the challenge of understanding how "mathematical concepts turn up in entirely unexpected connections" (Wigner 1960, p. 2). The second challenge is how we can "know whether a theory formulated in terms of mathematical concepts is uniquely appropriate" (p. 2), or what he later describes as the remarkable accuracy and (prima facie) explanatory power of false theories.

In this paper I shall explore these challenges by way of a detour through James Clerk Maxwell's "method of physical analogy." Maxwell most clearly articulates this methodology in his seminal 1855 article "On Faraday's Lines of Force", where he writes, "By a physical analogy I mean that partial similarity between the laws of one science and those of another which makes each of them illustrate the other. . . . [W]e find the same resemblance in mathematical form between two different phenomena" (Maxwell [1855] 1890, p. 156). Maxwell used this methodology repeatedly in his development of the theory of electromagnetism (see also Maxwell's "On Physical Lines of Force" (1861)), in particular by drawing physical analogies between fluid dynamics (hydrodynamics) and electromagnetic phenomena. So, for example, he conceives of Faraday's lines of force as "fine tubes of variable section carrying an incompressible fluid" (Maxwell [1855] 1890, p. 158). He notes that this is purely an "imaginary fluid" and ". . . not even a hypothetical fluid. . . . merely a collection of imaginary properties" (p. 160). His approach thus offers a case study in the use of fictional posits in science as well.

Maxwell does not simply employ these physical analogies and fictional posits with a naive opportunism, but rather engages in a philosophical reflection on both the legitimacy of such a methodology and its broader metaphysical implications. There are three points in Maxwell's philosophical reflections that I wish to call attention to and that are relevant for Wigner's challenges. The first concerns Maxwell's views on how mathematical models represent reality; second, his views on the explanatory power of mathematical models; and third, the version of scientific structuralism that Maxwell is led to, given the many successes of this method of physical analogy. I will very briefly outline these three points in what follows.

Maxwell conceives of his physical analogy as a middle path between what he calls a "purely mathematical formula" on the one hand and a "physical hypothesis" on the other. While traditionally the representational power of mathematics is conceived of as what might be described as a two-place relation[1]—the abstract mathematical equation and the physical system that is being represented—Maxwell often describes the representational power of mathematics as a three-place relation: the abstract mathematical equation "stripped of any physical dress" (Maxwell [1855] 1890, p.156; compare Hertz's [1893] 1962 "gay garment" remark, p. 28), the mathematical equation with a physical interpretation (what Maxwell calls an "embodied form"), and finally the physical system being modeled. One of the central points of Maxwell's physical analogy is that the physical interpretation or embodiment in this second kind of mathematical model need not be the same as the actual type of physical system being modeled in order to be scientifically fruitful and further our understanding. Maxwell for example writes, "my aim has been to present the mathematical ideas to the mind in an embodied form . . . not as mere symbols, which neither convey the same ideas, nor readily adapt themselves to the phenomena to be explained" (Maxwell [1855] 1890, p.187)

The suggestion in this last quotation is that it is only mathematics in an embodied form, which is suitably adapted for explanation. Again regarding a "purely mathematical formula" Maxwell writes "we entirely lose sight of the phenomena to be explained; and though we may trace out the consequences of given laws, we can never obtain more extended views of the connexions of the subject" (Maxwell [1855] 1890, p. 155). I think we often loose sight of this distinction between an equation in a purely abstract form and an embodied equation with a physical interpretation because in both cases we write down the exact same string of symbols. This distinction is both relevant to current debates about the explanatory power of mathematics and can help us make better sense of the various twists and turns in the history of science regarding the discoveries of such equations. Often times a scientist will find the right mathematical equation, without finding until much later the right physical interpretation of that equation, that we now take for granted when we look at the equation.

The third and final issue explores how Maxwell's views on physical analogies leads him to what is arguably a form of scientific structuralism. I use the term scientific structuralism broadly to encompass any structural approach to science, not just structural realism. Although I believe that Maxwell is a scientific realist broadly construed (though one who recognizes a legitimate function for fictions in the elucidation and explanation of phenomena) he is not quite a structural realist in the sense of thinking that the structural continuity across different fields of science that are exploited in his physical analogies, thereby licenses an inference to the reality of those structures. The relation between Maxwell's method of physical analogy and what I am calling his scientific structuralism is expressed most clearly in his 1856 essay "Are there Real Analogies in Nature?"

[1] I am somewhat uncomfortable in describing this as a relation, which tends to reify the relata, and it might be better described as three layers, or two kinds of mathematical models of varying abstraction.

Maxwell begins the essay by dismissing the obvious objection that "no question exists as to the possibility of an analogy without a mind to recognise it—that is rank nonsense" (Maxwell 1856 [1882], p. 236). But he continues,

> Now, if in examining the admitted truths in science and philosophy, we find certain general principles appearing throughout a vast range of subjects, and sometimes re-appearing in some quite distinct part of human knowledge . . . are we to conclude that these various departments of nature in which analogous laws exist, have a real inter-dependence; or that their relation is only apparent and owing to the necessary conditions of human thought? (Maxwell 1856 [1882], p. 236)

Maxwell opts for the former, rather than the latter Kantian answer. He elucidates these analogous laws in terms of the notion of relations:

> Although pairs of things may differ widely from each other, the *relation* in the one pair may be the same as that in the other. Now, as in a scientific point of view the *relation* is the most important thing to know, a knowledge of the one thing leads us a long way towards a knowledge of the other. (Maxwell 1856 [1882], p. 243 emphasis original)

This emphasis on the *relations* rather than the relata as being what is important for science, casts Maxwell as a structuralist (broadly conceived) and gives insight into why his method of physical analogies is so useful.

Maxwell's discussions of relations in nature, which undergird his method of physical analogy, brings us back to the challenges posed by Wigner at the outset, regarding the unreasonable effectiveness of mathematics. With regard to Wigner's second challenge, regarding the non-uniqueness of representation, explanatory power, and confirmation, we see Maxwell grappling with these same questions: In his seminal "On Physical Lines of Force" he notes,

> The explanation of any number of them [electro-magnetic facts] by several different hypotheses must be interesting, not only to physicists, but to all who desire to understand how much evidence the explanation of phenomena lends to the credibility of a theory, or how far we ought to regard a coincidence in the mathematical expression of two sets of phenomena as an indication that these phenomena are of the same kind. (Maxwell 1861 [1890], p. 488)

I will conclude by briefly drawing some implications from Maxwell's discussions for current approaches to solving Wigner's unreasonable effectiveness of mathematics.